DYNAMIC CHARACTERISTICS OF THERMOANEMOMETERS WITH GLASS-COVERED TRANSDUCERS

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This study deals with the frequency characteristics of a glass-covered thermistor serving as transducer in a thermoanemometer and of a constant-resistance thermoanemometer with such probes.

An evaluation of the measuring performance of thermoanemometers with probes protected against ambient hazards ranks among the most important engineering tasks related to modern methods of fluid flow analysis. In some studies concerning the use of metal and semiconductor-type thermistors for measurements of transient fluid flow no consideration was given to the effect of the insulating cover on the probe and the overall instrument characteristics [1, 2]. At the same time, all available test data prove that the presence of glass in thermistor probes used for semiconductor-type thermoanemometers [3] does appreciably affect the instrument characteristics.

The author has made an attempt to theoretically evaluate the characteristics of thermoanemometers with probes protected against the moving ambient medium by means of a layer of special insulation. In the first approximation, the physical pattern of heat transfer between fluid and thermoanemometer probe is described by a one-dimensional mathematical model, with the real transducer (a bead of temperature-sensitive material coated with glass) replaced by an infinitesimally thin hot film between two sheets of insulating material at the ordinate $y = 0$ (Fig. 1). The thickness of each glass layer is l; the mass of the sensing element can be taken into account in terms of the total heat capacity C_T determined from tests with a real thermistor.

In order to linearize the heat transfer equation, all variables are separated into average-in-time and fluctuation components (they are denoted by different symbols).

The equation of steady-state heat conduction through a glass layer is

$$\frac{\partial^2 T}{\partial y^2} = 0. \tag{1}$$

The boundary conditions for Eq. (1) are defined on the basis of the following concepts:

1. At the location of the hot film ($y = 0$) the steady-state temperature of the glass T is equal to the operating temperature of the film T_D and the thermal flux (heat dissipated per unit area of thermistor surface) is transmitted by conduction through the glass layers:

$$T = T_D;\ Q = -2\lambda \frac{\partial T}{\partial y}; \tag{2}$$

2. At the glass–fluid boundary ($y = l$), as a result of the temperature difference between glass and fluid, there occurs a heat transfer

$$-2\lambda \frac{\partial T}{\partial y} = A(T - T_L) \tag{3}$$

(the factor 2 in both boundary conditions accounts for the symmetry of heat conduction through the two glass layers).

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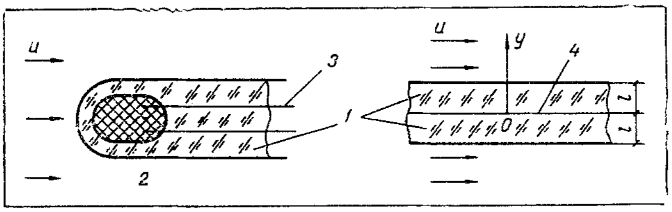


Fig. 1. MT-64 thermistor and the mathematical model of heat transfer between it and moving fluid: 1) glass which insulates the transducer from the fluid; 2) bead of heat sensitive semiconductor material; 3) thermistor leads; 4) hot film.

Inserting the solution to Eq. (1) with the boundary conditions (3) into the formula for the so-called total heat transfer at the probe, as if the latter were in direct contact with the moving fluid,

$$A_{tot} = \frac{Q}{T_D - T_L}, \tag{4}$$

we have

$$A_{tot} = \frac{A}{\frac{A}{2\lambda} l + 1}. \tag{5}$$

The temperature drop across the glass–fluid boundary is

$$\Delta T = T|_{y=l} - T_L = \frac{T_D - T_L}{1 + \frac{A}{2\lambda} l}. \tag{6}$$

The thermal flux from an electrically heated transducer element

$$Q = \frac{I^2 R}{S} \tag{7}$$

can be determined experimentally and the coefficient of steady-state heat transfer at the probe–fluid boundary can be found from (4) and (5).

The equation of transient heat transfer through a glass layer is, in operator form:

$$a \frac{\partial^2 \tau}{\partial y^2} = p\tau. \tag{8}$$

The boundary conditions for this equation are:

1. At the location of the hot film (y = 0) the fluctuating temperatures of the glass and of the transducer (and their models) are equal:

$$\tau = \tau_D. \tag{9}$$

2. The heat from the film is partly transferred to the glass layers and partly spent on increasing the internal energy of the transducer element, in transform notation,

$$q = -2\lambda \frac{\partial \tau}{\partial y} + C_T p\tau. \tag{10}$$

3. At the glass–fluid boundary (y = l) the incoming fluctuating thermal flux is transferred to the moving fluid. Applying the Laplace transformation to condition (3) after differentiation, we obtain

$$-2\lambda \frac{\partial \tau}{\partial y} = A\tau + \alpha (T - T_L). \tag{11}$$

Inserting the solution to Eq. (8) into expression (10) for the transform of the fluctuating thermal flux yields

$$q = \frac{2\lambda k \left[(A \operatorname{ch} kl + 2\lambda k \operatorname{sh} kl) \tau_D + \alpha \Delta T \right]}{A \operatorname{sh} kl + 2\lambda k \operatorname{ch} kl} + C_T p \tau_D, \tag{12}$$

with the frequency parameter

$$k = \sqrt{p/a}. \tag{13}$$

In this case of a thermoanemometer transducer, a Laplace transformation of Eq. (7) after differentiation yields

$$q = \frac{2IRi + I^2 r}{S}. \tag{14}$$

If one considers also that the following characteristic relation between resistance and temperature fluctuations applies to a thermistor

$$r = \alpha_\rho R \tau_D, \tag{15}$$

then

$$r\left\{\frac{1}{\alpha_\rho R}\left[\frac{2\lambda k(A\,\mathrm{ch}\,kl + 2\lambda k\,\mathrm{sh}\,kl)}{A\,\mathrm{sh}\,kl + 2\lambda k\,\mathrm{ch}\,kl} + pC_\mathrm{T}\right] - \frac{I^2}{S}\right\} = \frac{2IRi}{S} - \frac{2\lambda k\alpha\Delta T}{A\,\mathrm{sh}\,kl + 2\lambda k\,\mathrm{ch}\,kl}. \tag{16}$$

With no glass (l = 0), we obtain a transform of the well known heat balance equation for a thermoanemometer transducer [4]

$$pC_\mathrm{T}\tau + A\tau + \alpha\Delta T = \frac{2IRi + I^2 r}{S}, \tag{17}$$

and the fluctuations of thermistor resistance r are related to the fluctuations of electric current i and to the fluctuations of heat transfer α by the following equation in operator form

$$r\left[1 + p\frac{C_\mathrm{T}}{\alpha_\rho R\left(\frac{A}{\alpha_\rho R} - \frac{I^2}{S}\right)}\right] = \frac{\frac{2IR}{S}i - \frac{\Delta TS}{2IR}\alpha}{\frac{A}{\alpha_\rho R} - \frac{I^2}{S}} \tag{18}$$

which represents a first-order transfer function.

With thin insulating layers and at low frequencies ($kl \ll 1$) expression (16) reduces to an analogous one:

$$r\left[1 + p\frac{C_\mathrm{T}}{\alpha_\rho R\left(\frac{2\lambda}{\alpha_\rho lR} - \frac{I^2}{S}\right)}\right] = \frac{\frac{2IR}{S}\left[i - \frac{2\lambda}{l}\frac{\alpha}{A}\frac{\Delta TS}{2IR}\right]}{\frac{2\lambda}{\alpha_\rho lR} - \frac{I^2}{S}}. \tag{19}$$

At $kl > 1$ relation (16) is accurately enough approximated by the relation

$$r\left[\frac{I^2}{S} + \frac{2\lambda}{\alpha_\rho R}\sqrt{\frac{p}{a}} + \frac{C_\mathrm{T}}{\alpha_\rho R}p\right] = \frac{2\alpha\Delta T}{\left(1 + \frac{A}{2\lambda k}\right)\exp(kl)} - \frac{2IRi}{S}, \tag{20}$$

or

$$\frac{r}{2R} = \zeta W_\mathrm{T}\frac{\alpha}{A} - W_\mathrm{T}\frac{i}{I}, \tag{21}$$

where

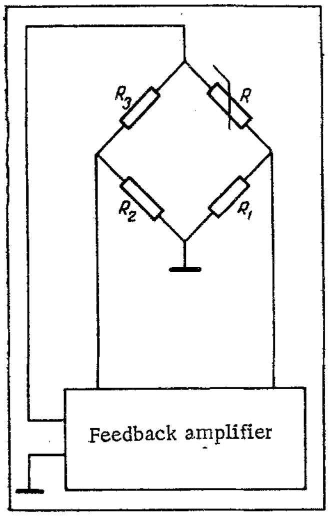
$$W_\mathrm{T} = \frac{1}{1 + \sqrt{pN'} + pM'};\quad \zeta = \frac{1}{\left(1 + \frac{A}{2\lambda k}\right)\exp(kl)} \tag{22}$$

and

$$N' = \left(\frac{2\lambda}{\alpha_\rho Q}\right)^2\frac{1}{a};\quad M' = \frac{C_\mathrm{T}}{\alpha_\rho Q} \tag{23}$$

are the thermistor time constants.

The transient equation (21) indicates that at high frequencies the responses of a glass-covered transducer to electric current fluctuations and to heat transfer (flow rate) fluctuations differ by a factor ζ.



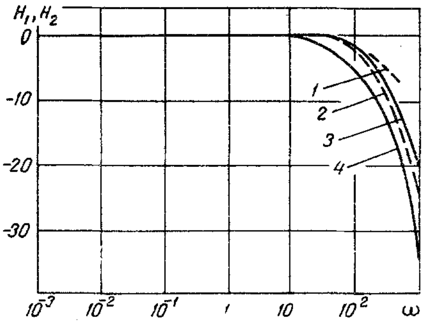


Fig. 2 Fig. 3

Fig. 2. Schematic block diagram of a constant-resistance thermoanemometer.

Fig. 3. Amplitude-frequency characteristics of a thermistor, calculated according to formula (16): 1) ratio of moduli in the expressions for current and resistance fluctuations in a transducer, at a water velocity 2 m/sec; 2) the same at a zero water velocity; 3) ratio of moduli in the expressions for heat transfer and resistance fluctuations in a transducer, at a water velocity 2 m/sec; 4) the same at a zero water velocity. H (dB).

When the constant-resistance principle is used in a thermoanemometer, the thermistor becomes one arm of a Wheatstone bridge and an unbalance signal from the latter is transmitted to the input of a feedback amplifier. The output from this amplifier is fed across the supply diagonal of the bridge circuit. When the conditions in the stream around the probe change, the electric current in the latter changes too and the transducer resistance remains almost constant.

With a feedback-amplifier gain K_V, a fluctuation of the transducer resistance r will change the transducer current by [6]

$$i=\frac{rI}{R+R_1}\left[\frac{R_1K_v}{R+R_1}-1\right] \tag{24}$$

i.e.,

$$\frac{i}{I}=\frac{r}{2R}\cdot\frac{2m}{m+1}\left(\frac{K_v}{1+m}-1\right), \tag{25}$$

where m denotes the ratio of thermistor resistance to resistance R_1 in series with it (Fig. 2).

Combining Eqs. (21) and (25) will yield the sought transfer function of a constant-resistance thermoanemometer with a glass-covered probe:

$$\frac{\frac{i}{I}}{\frac{\alpha}{A}}=\frac{1}{\left(1+\frac{A}{2\lambda k}\right)\exp(kl)\left(1+\sqrt{pN}+pM\right)}, \tag{26}$$

where the instrument time constants

$$N=\frac{N'(m+1)^4}{K_v^2(2m)^2};\quad M=\frac{M'(m+1)^2}{K_v 2m} \tag{27}$$

are smaller than the respective thermistor time constants N' and M' by an amount which depends on the feedback-amplifier gain.

The form of the exponential factor in (26) suggests that reducing the glass thickness l will appreciably improve the response characteristics of the instrument in the high-frequency range.

This simplified theory based on a one-dimensional model of a protected transducer-probe in a fluid stream yields, of course, an only rough approximation to the true spatial pattern, but it is nevertheless adequate for comparing the dynamic characteristics of thermoanemometers with glass-covered probes of various thicknesses. It becomes possible to evaluate the quality of measurements of fluctuating flow parameters when such measurements are made with these instruments.

Calculated frequency characteristics for a probe of a water thermoanemometer with MT-64 thermistors [3] are shown in Fig. 3. The following data were used in the computation: static calibration of the instrument $I^2 = 0.225 \cdot 10^{-3} + 0.02 \cdot 10^{-3}\sqrt{u}$, operating thermistor resistance $R = 585\ \Omega$, operating temperature $T_D = 331°K$, water temperature $T_L = 289°K$, temperature difference $T_D - T_L = 42°C$, temperature coefficient of resistance of the MT-64 at operating temperature $\alpha_\rho = -3.24 \cdot 10^{-2}/°C$, thermal capacity of thermistor $C_T = 44\ J/m^2 \cdot °C$, thermal conductivity of glass $\lambda = 1.255\ W/m \cdot °C$, thermal diffusivity of glass $a = 0.6 \cdot 10^{-6}\ m^2/sec$, outside probe radius $0.32 \cdot 10^{-3}$ m, estimated glass thickness $l = 0.14 \cdot 10^{-3}$ m, and active surface of heat transfer from transducer to glass $S = 0.203 \cdot 10^{-6}\ m^2$.

The curves plotted for flow velocities 0 and 2 m/sec indicate a distinct difference between the thermistor response to fluctuations of heat transfer α with the surrounding water and to fluctuations of transducer current; the probe response was defined as the fluctuation of resistance r according to Eq. (16). As is to be expected, the frequency characteristics of this thermistor improve rapidly with increasing stream velocity. An experimental check of the relation (26) for a semiconductor-type thermoanemometer with a feedback-amplifier gain $K_V = 43$ has shown that this relation as well as relation (16) remain valid within 30% at zero water velocity, which confirms that this model is suitable for purposes of evaluating the use of thermoanemometers with glass-covered probes in dynamic measurements.

NOTATION

- A is the average-in-time coefficient of heat transfer at the glass–fluid boundary, $W/m^2 \cdot °C$;
- A_{tot} is the coefficient of steady-state heat transfer at a bare probe (a fictitious quantity introduced for gauging the heat transfer between a glass-covered probe with the moving fluid), $W/m^2 \cdot °C$;
- a is the thermal diffusivity of glass which insulated the heat sensitive element from the fluid, m^2/sec;
- C_T is the total thermal capacity of transducer, $W \cdot sec/m^2 \cdot °C$;
- H_1 is the ratio of moduli in the expressions for current and resistance fluctuations in the transducer, dB;
- H_2 is the ratio of moduli in the expressions for heat transfer and resistance fluctuations in the transducer, dB;
- I is the quiescent current through thermistor, A;
- i is the transform of fluctuation current through thermistor, A;
- K_V is the voltage gain of feedback amplifier;
- k is the frequency parameter, 1/m;
- l is the thickness of glass layer, m;
- N' is the intrinsic time constant of thermistor, sec;
- N is the time constant of constant-resistance thermoanemometer, sec;
- M' is the intrinsic time constant of thermistor, sec;
- M is the time constant of constant-resistance thermoanemometer, sec;
- p is the complex variable in the Laplace transformation;
- Q is the average-in-time thermal flux from the transducer, W/m^2;
- q is the transform of thermal flux fluctuations in the transducer, W/m^2;
- R is the average-in-time operating resistance of thermistor, Ω;
- R_1 is the constant resistance in series with the thermistor in the thermoanemometer circuit, Ω;
- r is the transform of resistance fluctuations in the thermistor, Ω;
- S is the effective surface area of heat transfer from the transducer, m^2;
- T_D is the steady-state temperature of hot film, °K;
- T is the steady-state temperature of insulating glass layer, °K;
- T_L is the temperature of fluid, °K;
- u is the velocity of oncoming fluid, m/sec;
- W_T is the relation between resistance fluctuations and current in the thermistor, in operator form;
- y is the space coordinate in the mathematical model of the transducer, m;

α is the fluctuation component of heat transfer coefficient, $W/m^2 \cdot °C$;
α_ρ is the temperature coefficient of resistance, $1/°C$;
λ is the thermal conductivity of insulating material, $W/m \cdot °C$;
τ_D is the transform of temperature fluctuations in the hot film, °K;
τ is the transform of temperature fluctuations in the insulating glass layer, °K;
ζ is the coèfficient in the transfer function of a thermistor at high frequencies.

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